Lecture 4: Forces in Space

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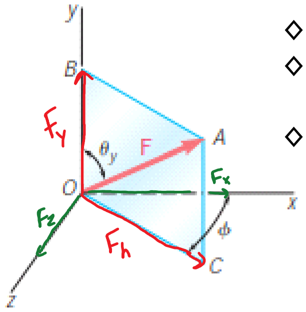
2.12 Rectangular components of a force in space

Vectors also exist in 3 dimensions. Let **F** represent a force (x,y,z) acting on the origin O:



- The plane that F lies on passes through the y-axis
- Its orientation is defined by the angle ϕ it forms with the xy plane
- The direction of F within the plane is defined by θ_y that it forms with the y-axis

We can separate F into its components:

$F_y = Fcos(\theta_y)$

$F_h = Fsin(\theta_y)$

But F_h can still be resolved to two components on the *xz* plane.

$F_x = F_h cos(\phi) = Fsin(\theta_y)\cos(\phi)$

$F_z = F_h sin(\phi) = Fsin(\theta_y)\sin(\phi)$

Therefore, we resolved the given force **F** into three components: F_x, F_y, F_z

Applying the Pythagorean Theorem:

$F^2 = (OA)^2 = (OB)^2 + (BA)^2 F_y^2 + F_h^2$

$F_h^2 = (OC)^2 = (DC)^2 = F_x^2 + F_z^2$

By subbing the second equation into the first, we can evaluate the magnitude of **F** using this formula:

$$F = \sqrt{F^2{}_x + F^2{}_y + F^2{}_z}$$

Additionally, we can use the angles between **F** and the x,y, and z axes, $\theta_x, \theta_y, and\ \theta_z$ to obtain:

$F_x = F\cos\theta_x$	$F_y = F\cos\theta_y$	$F_z = F\cos\theta_z$

These angles define the direction of *F*, and the cosines of them are called the **direction cosines** of the force **F**.

F can be broken up into unit vector components.

$F = F_x \mathrm{i} + F_y j + F_z k$

$F = F, \cos\theta_x \mathrm{i} + \cos\theta_y j + \cos\theta_z k$

$F = F\lambda$

where $\lambda = \cos\theta_x i + \cos\theta_y j + \cos\theta_z k$

|λ|=1, and same direction as **F**.

The components of λ are equal to the direction cosines of the line of action of **F**.

$\lambda_x = \cos\theta_x$

$\lambda_y = \cos\theta_y$

$\lambda_z = \cos\theta_z$

This means that:

$\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1$

and:

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

If a problem gives you the components of force **F**, we can determine its magnitude using the formula above, and from $F_x = F\cos\theta_x$ we can find the direction cosines.

$\cos\theta_x = \frac{F_x}{F}$	$\cos\theta_y = \frac{F_y}{F}$	$\cos\theta_z = \frac{F_z}{F}$

2.13 Force defined by its magnitude and two points on its line of action

Given **F** and two points on its line of action $M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$:

$MN = d_x \mathrm{i} + d_y j + d_z k$

The unit vector λ along **F** or MN can be found by dividing vector MN by its magnitude |MN|, or *d*: vector

$$\lambda = \frac{MN}{|MN|} = \frac{MN}{d} = \frac{1}{d}(d_x i + \mathrm{d}_y j + d_z k)$$

magnitude

$F = F\lambda$

$F = \frac{F}{d}(d_x i + d_y j + d_z k)$

$F_x = \frac{F\,\mathrm{d}_x}{d}$	$F_y = \frac{F\,\mathrm{d}_y}{d}$	$F_2 = \frac{F\,\mathrm{d}_z}{d}$

The component forces are proportional to the geometric ratio defining the position vector of *M* and *N*.

The component forces of **F** are now incredibly easy to find when we have 2 points *M* and *N*.

$d = \sqrt{d_x^2 + d_y^2 + d_z^2}$

$\mathrm{d}_x = x_2 - x_1$

$\mathrm{d}_y = y_2 - y_1$

$d_z = z_2 - z_1$

2.14 Addition of concurrent forces in space

Resolve each force into components.

$R_x = \sum F_x$	$R_y = \sum F_y$	$R_z = \sum F_z$

$|R| = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$\cos\theta_x = \frac{R_x}{R}$	$\cos\theta_y = \frac{R_y}{R}$	$\cos\theta_z = \frac{R_z}{R}$

2.15 Equilibrium of a particle in space

If a particle is in equilibrium, the resultant of the forces acting on it is zero.

$\sum F_x = 0$	$\sum F_y = 0$	$\sum F_z = 0$

^no more than 3 unknowns can be present when solving problems in equilibrium.

<u>Solving problems</u>:

1. Draw a free body diagram of the particle and all the forces acting on it
2. Use the 3 equations of equilibrium listed above to solve for the 3 unknowns.